Markscheme

May 2017

Mathematics

On-screen examination

Error Carried Forward (ECF) marks

Errors made at any step of a solution affect all working that follows. In general, **Error Carried Forward (ECF)** marks are awarded after an error.

    a)   **ECF** applies from one part of a question to a subsequent part of the question and also applies within the same part.

    b)   If an answer resulting from **ECF** is inappropriate (*eg*, negative distances or $\sin x > 1$) then subsequent marks should not be awarded.

    c)   If a question is transformed by an error into a **simpler question** then **ECF** may not be fully awarded.

    d)   To award **ECF** marks for a question part, **there must be working present for that part**.

    e)   **ECF** is only applied to working which is correct. This means that all working subsequent to an error must be checked for accuracy.

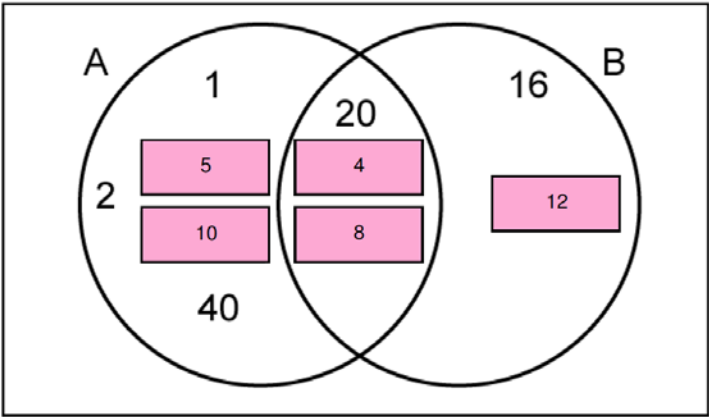   f)   A misread (**MR**) is an error**. ECF** is normally awarded.

General points

    a)   As this is an international examination, accept all alternative forms of **notation,** for example 1.9 and 1,9 **OR** 1 000 **OR** 1.000. However **DO NOT ACCEPT** incorrect mathematical notation e.g $x\verb|^|2$ for x^2 unless noted otherwise in the MS.

    b)   Ignore further working after a correct answer **unless** it indicates a lack of mathematical understanding **i.e. if the further working contradicts the correct answer,** then the last mark cannot be awarded.

    c)   Where candidates have written two solutions to a question, mark the response that deserves more marks.

    d)   In the markscheme, equivalent examples of **numerical** and **algebraic** forms or **simplified** answers will generally be written in the notes preceded by **OE** 'or equivalent' e.g. $\dfrac{1}{2}$ **OR** 1/2 **OR** $1 \div 2$ and $\dfrac{x}{2}$ **OR** $x/2$ **OR** $x \div 2$

    e)   In the markscheme, information provided in brackets indicate detail that may be seen in a candidate response but is not necessary to award the marks.

    f)   Special case marks **SC** can be allocated instead of but not in addition to the marks prescribed in the markscheme.

    g)   Accept seeing equations not in-line.

    h)   Accept notation errors in intermediate steps.

    i)   When a calculator screenshot is taken, accept not seeing the whole operation

Example 1
- 1 mark awarded and corresponding notes are aligned

b			
	• Show clear line of reasoning in the method	45 & 49 seen **OE** *e.g.* $49 = 45 + x$	**2**
	• 4	**ACCEPT** 45+X/10=4.9 **and** Ans 4	

Task 1

		Answers	**Notes**	**Total**
1	a	•¹ 2^3 **OR** 5 indicated as factor	•¹ seen as multiplied factor **OR** seen in division ACCEPT 2*2*2 **OR** 2^3 **OR** 2.2.2 DO NOT ACCEPT 2+2+2 DO NOT ACCEPT 5 in a list	**2**
		•² (40 =) 2^3x5	•² ACCEPT 2*2*2*5 **OR** 2^3 x 5 **OR** 2.2.2.5	
	b	•¹ 4, 5, 8, 10 •² 8, 12	For •¹ **and** •² ACCEPT any order	**2**
	c	1 mark for each fully correct set 	•¹ 5 , 10 •² 4 , 8 •³ 12 Mark part (c) independently from part (b). **ECF** from part (b) not allowed	**3**
	d	4 ,8, 20	**ECF** allowed from (c) **ECF** allowed from (b) only if (c) is NR Do not award the mark if extra numbers are listed	**1**

		Answers	Notes	Total
1	e	•¹ either multiples of 4 **OR** factors of 40 •² completely correct answer	•¹ ACCEPT divisible by 4 •² They are the multiples of 4 (less than 21) **and** factors of 40. "**and**" or WTTE must be seen for ex: "both" "as well as" **OR** "meanwhile" **OR** "also" Extra descriptions not contradicting the correct description can be ignored Description not in context ex: "They are elements of both sets A and B". Award 0 marks	**2**

		Answers	Notes	Total
2	a	(Modal interval is) 45-49	Accept 45 to 49 **OR** 45,49 **OR** 45≤x≤49 **OR** [45,49] Do not accept 45<x<49 **OR** [45,49[	**1**
	b	52	If more than one number is written then 0 marks	**1**
	c	•¹ Calculating IQR for medicine •² Calculating IQR for physics •³ Comment comparing the interquartile ranges Alternatively •¹ Recognizing LQ is the same •² Recognizing UQ in medicine is bigger •³ Comment comparing the interquartile ranges	•¹ (IQR for medicine=) 20 •² (IQR for physics=) 15 •³ IQR for physics is smaller than that of medicine **OE**: ex: difference of 5. IQR must be seen. Do not accept "range" •³ ACCEPT "The winners in physics are closer in age". •³ IQR of medicine must be bigger than physics regardless of their values **SC**: if both IQR are seen as 47 to 67 and 47 to 62 only, award only 1 mark for •¹ and •² then award •³ as appropriate	**3**
	d	•¹ identifying the frequency (55) •² dividing their frequency by 210	•¹ 55 identified **OR** seen as the numerator •¹ Do not accept 55 seen in a list for ex: 3+12+22+55+.. •² $\dfrac{(\text{their } 55)}{210}$ **OE** for example 0.262 **OR** 0.26 **OR** $\dfrac{11}{42}$ **OR** 26.2% **OR** 26% 55/210=0.3 Award 2 marks	**2**
	e	•¹ Applying the 20% to 210 •² Read off the graph the corresponding age	•¹ $\dfrac{20}{100} \times 210$ **OR** 42 **OR** $\dfrac{80}{100} \times 210$ **OR** 168	**2**

		Answers	Notes	Total
3	a	Step a: 14,3 . Step c: 5,7, 3 •¹ All step a correct •² All step c correct	•¹ 14, 3 •² *their* 5, 7, *their* 3 Allow **ECF** from *their* step a only if *their* step c is single digits	**2**
	b	•¹ Correct sum of *their* numbers •² Not divisible by 10 **OR** Not a multiple of 10	•¹ *their* 83 •² ACCEPT "must be a multiple of 10" NOTE: Sum their values correctly to a number divisible by 10 **and** saying it is valid because sum is divisible by 10 award •¹ only	**2**
	c	•¹ (61+ X =) 70 seen •² (X=) 9	 9 without working: award 2 marks	**2**

		Answers	Notes	Total
3	d	•¹ Instruction F correct	•¹ (If the doubled digit is a two-digit number) add the two digits together Accept "add the two numbers together" **OR** WTTE	
		•² Instruction G correct	•² Add the numbers Accept "plus" the numbers **OR** "sum" the numbers **OR** WTTE	
		•³ Instruction H correct	•³ If the sum is divisible by 10 then it is valid **OR** if the sum is not divisible by 10 then not valid **OR** check if (make sure) the number is divisible by 10 **OR** WTTE The instruction has to be in the correct place to award the mark **SC:** Step G adding and if sum divisible by 10 valid. Step H adding and if sum not divisible by 10 invalid Award only 1 mark for •² and •³ .Award •¹ as appropriate **SC:** Step G adding and if sum divisible by 10 valid Step H if the sum not divisible by 10 invalid Award 2 marks for •² and •³ .Award •¹ as appropriate	**3**

		Answers	Notes	Total
4	a	(f(4) =) 2		**1**
	b	•¹ substituting *their* 2 into 3f(x)-1	•¹ 3x(*their* 2) – 1 **OR** 6 -1	**2**
		•² *their* value of f(x) correct	•² *their* 5 5 without working award 2 marks **ATTENTION** If *their* value in (a) is not 2 allow **ECF** in (b) only if there is working	
	c	•¹ One correct algebraic step leading to a solution	•¹ $-2f(x) = 11 - 3$ **OR** $-2f(x) = 8$ •¹ Do not accept 3-2f(x)-11=0	**4**
		•² Dividing *their* equation by 2	•² $-f(x) = 4$	
		•³ *Their* f(x) calculated correctly	•³ $f(x) = -4$	
		•⁴ *Their* corresponding value of x correct	•⁴ x=0	

Task 2

		Answers	Notes		Total
5	a	3 (degrees)			1
	b	•¹ Setting Pythagoras **OR** finding angle BAE correctly	•¹ $AB^2 = 65^2 + 100^2$ *or angleBAE* $= 33.023...$accept not seeing this step		
		•² *Their* value of AB calculated correctly	•² $AB = \sqrt{14225}$ *or* 119.26286...		
		•³ Substituting *their* AB correctly into trigonometric ratio	•³ $\tan(their\,3) = \dfrac{BC}{theirAB}$ **OR** $\dfrac{BC}{\sin(their3)} = \dfrac{theirAB}{\sin(their90\text{-}3)}$		
		•⁴ Correct operation to calculate BC	•⁴$BC = \sqrt{14225}\ or\ 119.26286... \times \tan 3$ **OR** 6.2503...		
		6.25 AG			4
	c	•¹ Substituting 6.25 correctly into volume formula	•¹ $\dfrac{4}{3}\pi(6.25)^3$. Accept: 4/3*pi6.25^3		
		•² Volume calculated correctly using *their* substituted radius	•² $\dfrac{15625}{48}\pi$ **OR** *their* 1022.65 (using π) **OR** *their* 1022.135 (using 3.14)		
			•² Accept not seeing this step		
		•³ *Their* value correctly rounded to nearest m³	•³ 1023 (m³) **OR** 1022 (m³)		3

		Answers	Notes	Total
5	d	•¹ A valid mathematical reason •² Another valid reason (mathematical **OR** real-life)	Mathematical reasons: (i) Referring to rounding of measurements *or* calculations (ii) Referring to inner and outer diameter **OR** thickness of sphere Examples of real-life reasons: (i) Referring to shape not a perfect sphere (ii) Referring to the sphere is not totally filled with water (regardless the reason) (iii) Expansion of water DO NOT ACCEPT the vertical post may include water as well DO NOT ACCEPT referring to errors in measurements given DO NOT ACCEPT "because maybe I made mistakes"	**2**

5	e		**Marks**	**1**	**2**	
			(H) Calculate total water consumption for the households in the community	Two attempts from any of the calculations below: Dividing 300000 by 4 to find the number of households 300 000/4 (= 75 000) households **OR** Multiplying *their* 366 by *their* 75000 to find the amount of water consumed per day by households *their* 366 * *their* 75000 (= 27 450 000) litres per day **OR** Dividing their 366 by 24 to find the amount of water in litres per hour for each household *their* 366/24(=15.25) l/h **OR** Calculating how many litres for every household during the 4 hours outage time **OR** Acceptable estimation for the amount of water used by household per hour (between 1100 000 and 1200000) without calculations	Calculate correctly the amount of water used by households per hour (27 450 000/24 =) 1 143 750 litres per hour **OR** (15.25*75000=)1143750 litres per hour	**10**
			(L) Calculate total water consumption for individuals in the community	One attempt from any of the calculations below: Multiplying 300000 by *their* 51 to find the amount of water consumed per day by the individuals Ex: $300\,000 \times their\,51\,(=15\,300\,000)$ litres per day **OR** Dividing by 24 to find amount of water consumed per individual per hour *their* 51/24(=2.125) l/h **OR** Calculating how many litres for every individual during the 4 hours outage time **OR** Acceptable estimation for the amount of water used by individuals per hour (between 630000 and 650000 l/h) without calculations	Calculate correctly the amount of water used by individuals per hour Ex: (15300000/24=)637500 litres per hour **OR** (2.125*300000=)637500 litres per hour	

5	e		**Marks**	**1**	**2**
			(T) Estimate amount of time before the water runs out	Attempt to calculate estimate for time: Household $$\frac{950\,000}{their\,1\,143\,750}(\approx 0.830.....\text{hrs} \approx 50\text{ mins})$$ **Or** Individual $$\frac{950\,000}{their\,637\,500}(\approx 1.49.....\text{hrs} \approx 89\text{ mins})$$ **OR** attempt for household and individuals but wrong result $$\frac{950\,000}{their\,1\,143\,750 + their\,637\,500}(\approx not\,0.5)$$	Calculating *their* estimate combining both household and individual information correctly $$(\frac{950\,000}{their\,1\,143\,750 + their\,637\,500}) \approx 0.5\,(33.....)\text{hrs} \approx 30\,to\,35\text{ mins}$$
			(A) Advice for the community	Advice related to minimizing the use in general **OR** Advise related to minimize/stop one specific activity	Advice requesting clearly to stop/reduce at least two of the following activities that consume too much water: Showers, wash dishes, washing clothes, or garden watering
			(J) Justification of degree of accuracy	Weak justification not supported Examples: This is just an estimate **OR** We never know for sure the actual amount **OR** This is just an average **OR** Correct and sensible rounding for their value(s) of time Example 30 min or 0.5 hours Do not accept: "my time is very accurate because I made the correct calculations"	Good justification supported Examples Comment that the time calculated is if all population do not respond to advice and that the estimate of time can be more if they respond to advice **OR** Calculate time if they respond to advice **OR** the number of persons per household is an average so not accurate **OR** Referring to fact that during the specific 4 hours of outage not all activities considered in calculations are actually performed

NOTE: Seeing the total consumption for households and individuals (1 143 750 + 637500 =) 1781250 l/h allows H2 and L2

		Answers	Notes	Total
6	a	•¹ Substituting correctly into formula of area of circle	•¹ $\pi \times 10^2$ **OR** 100π **OR** 314	
		•² Calculating correctly *their* area of one sector	•² $\dfrac{1}{6}$ *their* area of circle seen **OR** 52.359.. (accept not seeing this step)	
		•³ Multiplying *their* area of sector by 3	•³ Area of sectors **OR** 3x *their* area of one sector **OR** $3 \times \dfrac{1}{6}$ area of circle **OR** $\dfrac{1}{2}$ area of circle **OR** 50π **OR** 157(.0796327)	
		•⁴ Subtracting *their* area of sectors from 173 correctly	•⁴ 173- *their* 50π **OR** 15.9(2036732 **OR** 16 (km²)) Award •⁴ only if *their* result is positive	**4**
	b	•¹ Calculating area of one sector **OR** area of circle	•¹ $\dfrac{1}{4}\pi \times 10^2$ **OR** $\pi \times 10^2$ **OR** 100π **OR** 314	
		•² Subtracting *their* area of four sectors from their area of square correctly	•² *their* $400 - their 100\pi$ **OR** $400 - 314$ **OR** 85.8(4073464) **OR** 86 (km²)	**2**

6	c		Marks	1	2		4
			(V) Comparing Values of part (a) and part (b)	Recognizing which of their values from part (a) and part (b) is bigger			
			(J) Justification	Weak justification Examples: -The area uncovered in the triangular formation is smaller -The smaller area uncovered means better signal -The area covered in triangular is only half a circle while the area covered in rectangular is a circle	Good justification Referring to the proportion of uncovered area to the total area		
			(P) Proposing formation	Proposing the formation in part A **OR** triangular formation as the most suitable formation.			

Task 3

		Answers	Notes	Total
7	a	•¹ Denominator •² Increased by 2	•¹ Do not accept "the lower term" **OR** "the bottom" **OR** "the down" •² ACCEPT adds 2 **OR** multiples of 2 **OR** even numbers **SC** award 1 mark for the following: Denominator multiple of $2k$ Denominator increases	**2**
	b	$\dfrac{1}{12}$ (when $k=6$)	Answer may be seen in the table attached to 7a only if no response in the box of 7b **NOTE** if their value in the table in 7a not same as response in 7b, mark response in 7b.	**1**
	c	$(x_A =)\dfrac{1}{2k}$	Accept $1/2k$ **OR** $(2k)^{-1}$ Do not accept $1/2xk$ **OR** $1/2*k$	**1**
	d	•¹ substituting a number $k \geq 7$ into their rule from (c) •² Refer to predictionn for $k \geq 7$ from table in part (a) •³ acknowledging that the two values above are equal	•¹ $\dfrac{1}{2\times 7}$. Accept $2\times 7 =14$ •² when $k=7$ in the table X_A=1/14 •³ They are equal **SC** if "tested" correctly award 1 mark Tested correctly is when they apply the steps of verification mentioned in the left column on a value of $k \leq 6$	**3**

Answers	Notes	Total

7 **e**

x coordinate of point B (x_B)	$\dfrac{-1}{2}$	$\dfrac{-1}{4}$	$\dfrac{-1}{6}$	$\dfrac{-1}{8}$
y coordinate of point B (y_B)	$\dfrac{1}{4}$	$\dfrac{1}{8}$	$\dfrac{1}{12}$	$\dfrac{1}{16}$
y coordinate of point C (y_C)	$\dfrac{-1}{4}$	$\dfrac{-1}{8}$	$\dfrac{-1}{12}$	$\dfrac{-1}{16}$

•¹ One correct column **OR** two correct rows

•² Fully correct

2

		Answers	Notes	Total
7	f	$\bullet^1$ one correct pattern described for Y_A $\bullet^2$ another correct pattern described for Y_A $\bullet^3$ one correct pattern described for X_B $\bullet^4$ another correct pattern described for X_B	$\bullet^1$ and $\bullet^2$ Examples of accepted described patterns for Y_A: Denominators are multiples of 4 **OR** divisible by 4 Denominator is 4 times k Y_A is the same as Y_B Y_A is opposite to Y_C **OR** same as Y_C but different sign Numerator is constant **OR** numerator=1 $Y_A= \dfrac{1}{4k}$ Do not accept denominator Y_A multiples of 2 **OR** even numbers Do not accept Y_A is always positive Do not accept Y_A multiples of 4 **OR** even numbers Do not accept the <u>bottom</u> is multiples of 4 $\bullet^3$ and $\bullet^4$ *Examples of accepted described patterns for X_B :* Denominators are multiples of 2 **OR** even numbers X_B is same as X_A but opposite sign Numerator is always -1 Denominator is double k $X_B=-X_A$ $X_B= -\dfrac{1}{2k}$ Do not accept X_B multiples of 2 **OR** even numbers Do not accept X_B is always negative	**4**

7	g	Mark	Predictions (P)	Description (D)	Testing (T)	Verifying (V)	Justify/proof (J)	Notation and terminology (N)	Communication (C)		22
		1	Correctly predicted one term when $k \geq 5$	Attempted to describe a pattern Ex: k increases by 1 **OR** Numerator is equal 1 **OR** Bottom is times 4	Attempted to test their described pattern Ex: Substitute into their formula using k $k \leq 4$	Attempted to verify their described pattern **OR** general rule using one value for $k \geq 5$ (ex: substitutes in their formula $k \geq 5$)	Attempted to justify their described pattern **OR** general rule Ex: Referring to denominator being in arithmetic sequence **OR** It is the opposite of YA **OR** opposite of YB **OR** Testing at least 2 values correctly and saying it is the same **OR** the denominator is always multiplied by 4 **OR** times 4	The notation and terminology have significant errors	No communication Only calculations **OR** algebraic steps		
		2	Correctly predicted two terms when $k \geq 5$	Correctly described pattern for their Yc in words Ex: Denominator is always k times 4 **OR** denominator increases by 4	Tested correctly their described pattern **OR** general rule using $k \leq 4$ By Comparing with their predicted value	Calculates correctly their value for $k \geq 5$ and mentions the corresponding value in the table	Correctly justify their general rule Ex: Seeing Denominator is divisible by 4 (or multiple of 4) is justification **OR** Attempt to substitute X_A **OR** Y_A in terms of k into the gradient formula **OE OR** referring to denominator being arithmetic sequence and attempt to write U_1 and d	The notation and terminology are mostly correct Award only if D3 is achieved	Weak communication		

| 7 | g | 3 | Correctly predicted Y_C up to $k=5$ | Attempted to describe pattern for their Yc as general rule Examples: Yc=4k

OR
The pattern is
$-\dfrac{1}{4k}$ | | Comment comparing the values above to verify | Attempt to prove the general rule for Yc Substitute X_A and Y_A in terms of k into the gradient formula OE correctly
OR
referring to denominator being arithmetic sequence and mentioning U_1 and d correctly | | Good communication

Award only if J2 is achieved | | |
| | | 4 | Correctly predicted all terms of Y_C up to $k=6$ | Correctly described pattern for their Yc as general rule
$y_C = -\dfrac{1}{4k}$ | | | Correctly prove the general rule for Yc Simplify correctly to reach the rule for Y_C
OR
Finding Un of the arithmetic sequence | | | | |

Exemplification for (7g):

Predictions (P): can be seen in the table at 7e **even if response box in 7g is empty**

k	1	2	3	4	5	6
x coordinate of point A (x_A)	$\dfrac{1}{2}$	$\dfrac{1}{4}$	$\dfrac{1}{6}$	$\dfrac{1}{8}$	$\dfrac{1}{10}$	$\dfrac{1}{12}$
y coordinate of point A (y_A)	$\dfrac{1}{4}$	$\dfrac{1}{8}$	$\dfrac{1}{12}$	$\dfrac{1}{16}$	$\dfrac{1}{20}$	$\dfrac{1}{24}$
x coordinate of point B (x_B)	$\dfrac{-1}{2}$	$\dfrac{-1}{4}$	$\dfrac{-1}{6}$	$\dfrac{-1}{8}$	$\dfrac{-1}{10}$	$\dfrac{-1}{12}$
y coordinate of point B (y_B)	$\dfrac{1}{4}$	$\dfrac{1}{8}$	$\dfrac{1}{12}$	$\dfrac{1}{16}$	$\dfrac{1}{20}$	$\dfrac{1}{24}$
y coordinate of point C (y_C)	$\dfrac{-1}{4}$	$\dfrac{-1}{8}$	$\dfrac{-1}{12}$	$\dfrac{-1}{16}$	$\dfrac{-1}{20}$	$\dfrac{-1}{24}$

Describing the pattern: Denominator arithmetic sequence with first term -4 and common difference -4 **OR** Denominator multiples of 4 with negative sign **OR** equivalent.

General rule $y_C = -\dfrac{1}{4k}$

Testing the rule using any value of k between 1 and 5, e.g.: $k = 3$, $y_C = -\dfrac{1}{12}$ and $4 \times 3 = 12$

Verifying the rule by one value of k beyond k=5, e.g.:

When k=6 $y_C = -\dfrac{1}{24} = -\dfrac{1}{4 \times 6}$, or $24 = 4 \times 6$ and my value in the table is 1/24 , they are equal

Proving the rule:
$$\frac{y_A - y_c}{x_A - x_c} = \frac{k(x_A)^2 - y_c}{x_A - 0} = 1$$
$$y_c = k(x_A)^2 - x_A = k\left(\frac{1}{2k}\right)^2 - \frac{1}{2k} = \frac{k}{4k^2} - \frac{1}{2k} = -\frac{1}{4k}$$